Accurate solution of eigenvalue problems in industrial life-cycle simulations

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Topics of presentation

- Introduction
  Life-cycle simulation, eigenvalue solutions and evolution

- Life-cycle simulation eigenvalue problems in NASTRAN
  Rotor dynamics – internal rotating components
  Vibro-acoustic analysis – air enclosed by structure
  Aero-elastic solutions – external air flow around structure
  Fluid-structure interaction – fluid enclosed or surrounding

- Conclusions
  Statistics, future requirements and predictions
Life-cycle simulation

- Phases of product life-cycle
  - Development phase – concept design, detail design
  - Manufacturing phase – prototype and production
  - Operational phase – usage, maintenance, recycling

- Goals of life-cycle simulation
  - Prediction and optimization of operational behavior
  - Analysis of product and validation of predictions

- Foci of lifecycle simulation
  - High fidelity mathematical models of physical phenomena
  - Simulate the interaction with the operational environment
Eigenvalue problems and solutions

- Characteristics of practical life-cycle eigenvalue problems
  - Complex and real, symmetric and unsymmetric, linear and quadratic
  - Wide frequency range of interest with interior eigenvalue solutions
  - Very large problem sizes requiring out of core algorithms

- Strategies for accurate eigenvalue solution of life-cycle simulations
  - Computational simplification of the mathematical model (symmetry)
  - Numerical balancing of the various physical components (scale)
  - Algorithmic improvements for uniform accuracy in range (shift)
  - Performance gain from physics specific operators (reduced order)
The mid 1970s: stick models

- Power methods
- 5,752 node points
- 2,108 finite elements
- 28,924 degrees of freedom
- 4 eigenvectors
- 2,679 CPU seconds
- 1.1 hours elapsed time
- 1 million words of memory
- 36 million words of disk space

\[ (K - \lambda_s M) \mu \varphi = M \varphi \]
The mid 1980s: car frames

- Reduction methods
- ~50,000 node points
- ~60,000 finite elements
- ~264,000 degrees of freedom
- 50 eigenvectors
- 2,505 CPU seconds
- 0.9 hours elapsed time
- 60 Mwords of memory
- 173 Mwords of disk space

\[ A = C^{-1}(M) \quad C^{-T}; \quad (K + \lambda_s M) = CC^T \]
The mid 1990s: full body models

- Block, shifted Lanczos method
- \(~ 275,000 \) elements,
- \(~ 270,000 \) node points
- \(~ 1.6 \) million degrees of freedom
- \(~ 1,000 \) eigenvectors
- \( 4,936 \) CPU seconds
- \( 221 \) GBytes of I/O
- \( 1.7 \) hours elapsed time
- \( 128 \) MWords of memory
- \( 65 \) GBytes of disk used

\[ Q_{j+1}B_{j+1} = (K-\lambda_j M)^{-1}MQ_j - Q_jA_j - Q_{j-1}B_j^T \]
The mid 2000’s: detailed models

- Domain decomposed solutions
- ~12 million node points
- ~7.2 million elements
- ~35 million degrees of freedom
- 20 eigenvectors
- ~16 GB memory used
- ~680 minutes of elapsed time
- ~100,000 CPU seconds
- ~11.5 Tera-bytes of I/O
- ~630 Giga-bytes of disk used
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Simulations and solutions

Rotational dynamics
  Sequence of quadratic, asymmetric eigenvalue problems
  Linearization and spectral transformation
Vibro-acoustics
  Quadratic, topologically unsymmetric eigenvalue problem
  Symmetrization and reduced order Lanczos method
Aero-elasticity
  Sequence of complex, quadratic eigenvalue problems
  Dynamic reduction and augmentation of modal space
Fluid-structure interaction
  Real, symmetric problem with dense mass
  Virtual mass and transformed inner product Lanczos method
Rotational dynamic simulation

- Applications
  - Turbines and power generators
  - Car wheels and landing gears
  - Windmills and rotating machinery
  - Drive trains

- Analyses
  - Whirl modes and critical speeds
  - Regions of instability and resonance
  - Frequency and transient response
Quadratic problem linearization

- Rotor dynamic equation of motion in fixed reference system
  \[ M \ddot{u} + (B + \Omega C) \dot{u} + (K + \Omega H) u = 0 \]

- Block linear form
  \[
  \begin{bmatrix}
    M & 0 \\ 0 & I
  \end{bmatrix} \begin{bmatrix}
    \ddot{u} \\ \dot{u}
  \end{bmatrix} + \begin{bmatrix}
    B & K \\ -I & 0
  \end{bmatrix} \begin{bmatrix}
    \dot{u} \\ u
  \end{bmatrix} = 0
  \]

- Transformation to frequency domain
  \[ u = e^{\lambda t} \phi \]
  \[ (\lambda^2 M + \lambda B + K) \phi = 0 \]

- Unsymmetric eigenvalue problem
  \[
  \begin{bmatrix}
    \psi \\ \psi
  \end{bmatrix}^T (\lambda \begin{bmatrix}
    M & 0 \\ 0 & I
  \end{bmatrix} + \begin{bmatrix}
    B & K \\ -I & 0
  \end{bmatrix}) \begin{bmatrix}
    \phi \\ \phi
  \end{bmatrix} = 0
  \]
  \[ (\lambda \begin{bmatrix}
    M & 0 \\ 0 & I
  \end{bmatrix} + \begin{bmatrix}
    B & K \\ -I & 0
  \end{bmatrix}) \begin{bmatrix}
    \phi
  \end{bmatrix} = 0 \]

- Complex solutions
  \[ \lambda_j = \alpha_j \pm i \omega_j \]
  \[ f_j = \frac{\omega_j}{2\pi} \]
  \[ g_j = 2 \frac{\alpha_j}{\omega_j} \]
Complex spectral transformation

- **Shifted eigenvalue**
  \[ \lambda = \lambda_s + \mu \]

- **Shifted equation**
  \[
  ( \mu \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \bar{B} + M \lambda_s & K \\ -I & \lambda_s I \end{bmatrix} ) \begin{bmatrix} \phi \\ \phi \end{bmatrix} = 0
  \]

- **Inverted eigenvalue**
  \[ \overline{\lambda} = \frac{1}{\mu} \]

- **Transformed problem**
  \[
  ( \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} + \overline{\lambda} \begin{bmatrix} \bar{B} + M \lambda_s & K \\ -I & \lambda_s I \end{bmatrix} ) \begin{bmatrix} \phi \\ \phi \end{bmatrix} = 0
  \]

- **Canonical form**
  \[
  \overline{A} \overline{\phi}_j = \overline{\lambda}_j \overline{\phi}_j \\
  \overline{\psi}_j^H \overline{A} = \overline{\lambda}_j \overline{\psi}_j^H \\
  \overline{A} = \begin{bmatrix} - \left( \bar{B} + M \lambda_s \right) & -K \\ I & -\lambda_s I \end{bmatrix}^{-1} \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}
  \]

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Eigenvalue results in rotor dynamics

Resonances, instabilities, responses

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Vibro-acoustic analyses

- Applications
  - Automotive vehicle interior noise
  - Aero launch systems acoustics
  - Airplane cabin acoustics

- Analysis components
  - Find structure’s interior surface
  - Mesh interior volume
  - Couple the surface loads

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Vibro-acoustic coupling surface

Missing on windshield and trunk lid

Still missing on side door

Normal tolerance = .2

Normal tolerance = .5

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Coupled form symmetrization

- **Vibro-acoustic equilibrium**
  \[
  \begin{bmatrix}
  M_s & 0 \\
  A & M_f
  \end{bmatrix}
  \begin{bmatrix}
  \ddot{u} \\
  \ddot{p}
  \end{bmatrix}
  +
  \begin{bmatrix}
  K_s & -A^T \\
  0 & K_f
  \end{bmatrix}
  \begin{bmatrix}
  u \\
  p
  \end{bmatrix}
  =
  \begin{bmatrix}
  0 \\
  0
  \end{bmatrix}
  \]

- **First equation**
  \[
  M_s \ddot{u} + (K_s + A^T M_f^{-1} A)u - A^T (p + M_f^{-1} A u) = 0
  \]

- **Second equation**
  \[
  M_f K_f^{-1} M_f (\ddot{p} + M_f^{-1} A \ddot{u}) - Au + M_f (p + M_f^{-1} A u) = 0
  \]

- **Balancing**
  \[
  q = p + M_f^{-1} A u
  \]

- **Symmetric equation**
  \[
  \begin{bmatrix}
  M_s & 0 \\
  0 & M_f K_f^{-1} M_f
  \end{bmatrix}
  \begin{bmatrix}
  \ddot{u} \\
  \ddot{q}
  \end{bmatrix}
  +
  \begin{bmatrix}
  K_s + A^T M_f^{-1} A & -A^T \\
  -A & M_f
  \end{bmatrix}
  \begin{bmatrix}
  u \\
  q
  \end{bmatrix}
  =
  \begin{bmatrix}
  0 \\
  0
  \end{bmatrix}
  \]
Reduced order Lanczos method

- Partitioning
  \[
  \begin{bmatrix}
  P^T \\
  P_0^T
  \end{bmatrix}
  M
  \begin{bmatrix}
  P & P_0
  \end{bmatrix}
  =
  \begin{bmatrix}
  M & 0 \\
  0 & 0
  \end{bmatrix}
  \bar{Q}_j = P^T Q_j
  \]

- New operator
  \[
  \bar{A} = (K - \lambda_s M)^{-1} P \bar{M} \\
  \bar{Z}_j = \bar{A} \bar{Q}_j
  \]

- New recurrence
  \[
  \bar{Q}_{j+1}B_{j+1} = P^T \bar{Z}_j - \bar{Q}_j A_j - \bar{Q}_{j-1} B_j^T
  \]

- Mass orthogonal
  \[
  \bar{Q}_i^T \bar{M} \bar{Q}_j = Q_i^T P \bar{M} P^T Q_j = Q_i^T M Q_j = \delta_{ij}
  \]

- Spectrum retained
  \[
  \bar{Q}_j^T M \bar{P}^T \bar{A} \bar{Q}_j = Q_j^T M (K - \lambda_s M)^{-1} M Q_j = A_j
  \]

- Reduced cost
  \[
  \bar{Q}_{j+1} = \bar{Q}_{j+1} - \sum_i \bar{Q}_i \bar{Q}_i^T M \bar{Q}_{j+1}
  \]

- Vector recovery
  \[
  \varphi = \begin{bmatrix}
  P & P_0
  \end{bmatrix}
  \begin{bmatrix}
  I \\
  - (P_0^T K P_0)^{-1} (P_0^T K P)
  \end{bmatrix}
  \bar{\varphi}
  \]
Acoustic analysis results

Location 1

Location 2

Benchmark
NORML = .2
NORML = .5
NORML = .75

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Aero-elasticity

- Analyses
  - Static aero-elasticity
  - Aeroelastic optimization
  - Flutter analysis (effect of the unsteady, air-flow velocity dependent pressure distribution on the lifting surfaces)
- Aerodynamic theories
  - Lifting, strip, piston and interference theories
  - Subsonic and supersonic aerodynamic solutions
Dynamic reduction

- Analysis (boundary) and omitted (interior) partitions

\[
M_{ff} \ddot{u}_f + B_{ff} \dot{u}_f + K_{ff} u_f = \begin{bmatrix} M_{oo} & M_{oa} \\ M_{ao} & M_{aa} \end{bmatrix} \begin{bmatrix} \ddot{u}_o \\ \ddot{u}_a \end{bmatrix} + \begin{bmatrix} B_{oo} & B_{oa} \\ B_{ao} & B_{aa} \end{bmatrix} \begin{bmatrix} \dot{u}_o \\ \dot{u}_a \end{bmatrix} + \begin{bmatrix} K_{oo} & K_{oa} \\ K_{ao} & K_{aa} \end{bmatrix} \begin{bmatrix} u_o \\ u_a \end{bmatrix} = 0
\]

- Fixed boundary normal modes and static condensation modes

\[
K_{oo} \Phi_{om} = M_{oo} \Phi_{om} \Lambda_{mm} \quad K_{ao} G_{oa} = 0 \quad G_{oa} = -K_{oo}^{-1} K_{oa}
\]

- Transformation matrix and displacement transformation

\[
T_{fh} = \begin{bmatrix} \Phi_{om} & G_{oa} \\ 0 & I_{aa} \end{bmatrix} \quad u_f = \begin{bmatrix} u_o \\ u_a \end{bmatrix} = T_{fh} u_h \quad Q_{hh} = \begin{bmatrix} 0 & 0 \\ 0 & Q_{aa} \end{bmatrix}
\]

- Flutter problem in dynamically reduced space

\[
M_{hh} \ddot{u}_h + B_{hh} \dot{u}_h + ( K_{hh} - \frac{1}{2} \rho v^2 Q_{hh}(m,k) ) u_h = 0 \quad u_h = e^{i \omega t} \phi_h
\]

\[
( -\omega^2 M_{hh} + i \omega B_{hh} + K_{hh} - \frac{1}{2} \rho v^2 Q_{hh}(m,k) ) \phi_h = 0
\]

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Augmenting the modal space

- **Full modal space**
  \[ \Lambda_{oo} = \Phi_{oo}^T K_{oo} \Phi_{oo} \]
  \[ K_{oo}^{-1} = \Phi_{oo} \Lambda_{oo}^{-1} \Phi_{oo}^T \]

- **Complete flexibility**

- **Incomplete modal space**
  \[ \Phi_{oo} = [\Phi_{om} \quad \Phi_{or}] ; m << o, \ m + r = o \]

- **Component flexibilities**
  \[ K_{oo}^{-1} = \Phi_{om} \Lambda_{mm}^{-1} \Phi_{om}^T + \Phi_{or} \Lambda_{rr}^{-1} \Phi_{or}^T = Z_m + Z_r \]

- **Residual flexibility**
  \[ Z_r = K_{oo}^{-1} - \Phi_{om} \Lambda_{mm}^{-1} \Phi_{om}^T \]

- **Residual vectors**
  \[ \Psi_{ol} = Z_r G_o; P_o = G_o H_o(t) \]

- **Augmented space**
  \[ \Phi_{om} = [\Phi_{om} \quad \Psi_{ol}] ; \overline{m} = m + l \]
  \[ \Phi_{om}^T M_{oo} \Phi_{om} = I_{\overline{m} \overline{m}} \]
Eigenvalue results in flutter analysis

Frequency variation

Instability detection
Fluid structure interaction

- For structures that vibrate in fluid
- Fluid contained in a structure
- Fuel tanks
- Off-shore structures in water
- Incompressible fluids
- Low frequencies
- Virtual mass matrix approach
Virtual mass matrix

- Coupled vibration with incompressible fluid
  \[
  \begin{bmatrix}
  M_s & 0 \\
  A & 0
  \end{bmatrix}
  \begin{bmatrix}
  \ddot{u} \\
  \dot{p}
  \end{bmatrix}
  +
  \begin{bmatrix}
  K_s & -A^T \\
  0 & K_f
  \end{bmatrix}
  \begin{bmatrix}
  u \\
  p
  \end{bmatrix}
  =
  \begin{bmatrix}
  0 \\
  0
  \end{bmatrix}
  \]

- Eliminate pressure
  \[A\ddot{u} + K_f p = 0; \quad p = -K_f^{-1} A\ddot{u}\]

- First equation
  \[M_s \ddot{u} + K_s u - A^T p = M_s \ddot{u} + K_s u + A^T K_f^{-1} A\ddot{u} = 0\]

- Introducing virtual mass
  \[M_v = A^T K_f^{-1} A\]

- Symmetric equation of coupled equilibrium
  \[(M_s + M_v)\ddot{u} + K_s u = 0\]
Transformed inner product Lanczos

- Factorize mass
  \[ M = C C^T ; \quad \overline{Q}_j = C^T Q_j \]

- Transformed operator
  \[ \overline{A} = ( K - \lambda_s M )^{-1} C \]

- Transformed solution step
  \[ Z_j = \overline{A} \overline{Q}_j ; \quad \overline{Q}_j = C^T Q_j \]

- Lanczos recurrence step
  \[ \overline{Q}_{j+1} B_{j+1} = C^T Z_j - \overline{Q}_j A_j - \overline{Q}_{j-1} B_j^T \]

- Reduced cost
  \[ \overline{Q}_{j+1} = \overline{Q}_{j+1} - \sum_i \overline{Q}_i ( \overline{Q}_i^T \overline{Q}_{j+1} ) \]

- Spectrum retained
  \[ \overline{Q}_j^T C^T \overline{A} \overline{Q}_j = Q_j^T C C^T ( K - \lambda_s M )^{-1} C C^T Q_j \]
  \[ = Q_j^T M ( K - \lambda_s M )^{-1} M Q_j = A_j \]
Fluid-structure interaction results

With Fuel

No Fuel

Mode 1 – 7.5 Hz

Mode 1 – 27 Hz
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- Conclusions
  Statistics, state of the art and future predictions
Simulation matrix statistics

The constrained stiffness matrix of a current analysis problem:
- Number of rows: 35,734,709
- Nonzero terms: 1,384,305,995
- Nonzero terms in sparse factor matrix: 43,827,004,000
- Memory used during factorization: 1,080,732,000 (4 byte) words
- Actual elapsed time of sparse factorization on a high performance workstation: 335 minutes
Computational technology statistics

- Normal modes below 400 Hz
- ~268,000 node points
- ~275,000 finite elements
- 1.6 Million degrees of freedom
- Elapsed time and speed-up

- 64 node linux cluster
- Dual core (1.85 GHz) CPUs
- 50GB local SATA disks per node
- 4 GB memory per node
- GigE interconnect with HP MPI
State of the art in life-cycle simulation

- Detailed multi-disciplinary solution
- ~35 million node points
- ~34 million elements
- ~200 million degrees of freedom
- 1000 Hz frequency range
- ~20 GB memory used
- ~420 minutes of elapsed time
- ~52,000 CPU seconds
- ~4.2 Tera-bytes of I/O
- ~980 Giga-bytes of disk used
Future trends

- **Accurate solution** of eigenvalue problems will continue to constitute a crucial component of life-cycle simulations.

- Eigenvalue problem sizes continue to evolve and very likely to reach or even exceed the billion degree of freedom range: **Giga DOF problem**

- The ever shortening product development cycle time span in the industry require very highly scalable distributed techniques: **Peta-Scale computing**
Future requirements

▶ To extend the dominance of Lanczos method suggests more **physics sensitive adjustments** to the recurrence process for efficiency and accuracy reasons.

▶ The coupled nature of life-cycle phenomena desires methods for **sequences of eigenvalue problems** or the possibility of parametric eigenvalue solutions.

▶ The current **shifting and bounding technology** on the complex plane to establish regions containing clusters of eigenvalues needs significant improvement.
Thank You for your attention! Questions?