Accurate solution of eigenvalue problems in industrial life-cycle simulations

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Eigenvalue Problems
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Louis Komzsik
Chief Numerical Analyst
Office of Architecture and Technology
Siemens, A&C, A&D, PLM Software
Cypress, California

Topics of presentation

- **Introduction**

  Life-cycle simulation, eigenvalue solutions and evolution

- Life-cycle simulation eigenvalue problems in NASTRAN

  Rotor dynamics – internal rotating components

  Vibro-acoustic analysis – air enclosed by structure

  Aero-elastic solutions – external air flow around structure

  Fluid-structure interaction – fluid enclosed or surrounding

- Conclusions

  Statistics, future requirements and predictions

Life-cycle simulation

- Phases of product life-cycle
  - Development phase – concept design, detail design
  - Manufacturing phase – prototype and production
  - Operational phase – usage, maintenance, recycling
- Goals of life-cycle simulation
  - Prediction and optimization of operational behavior
  - Analysis of product and validation of predictions
- Foci of lifecycle simulation
  - High fidelity mathematical models of physical phenomena
  - Simulate the interaction with the operational environment

Eigenvalue problems and solutions

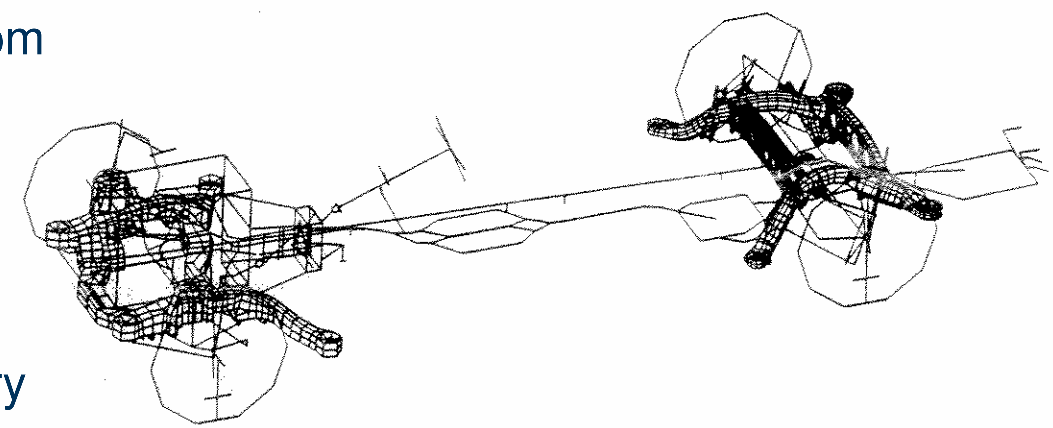
- Characteristics of practical life-cycle eigenvalue problems
  - Complex and real, symmetric and unsymmetric, linear and quadratic
  - Wide frequency range of interest with interior eigenvalue solutions
  - Very large problem sizes requiring out of core algorithms

- Strategies for accurate eigenvalue solution of life-cycle simulations
  - Computational simplification of the mathematical model (symmetry)
  - Numerical balancing of the various physical components (scale)
  - Algorithmic improvements for uniform accuracy in range (shift)
  - Performance gain from physics specific operators (reduced order)

The mid 1970s: stick models

- Power methods
- 5,752 node points
- 2,108 finite elements
- 28,924 degrees of freedom
- 4 eigenvectors
- 2,679 CPU seconds
- 1.1 hours elapsed time
- 1 million words of memory
- 36 million words of disk space

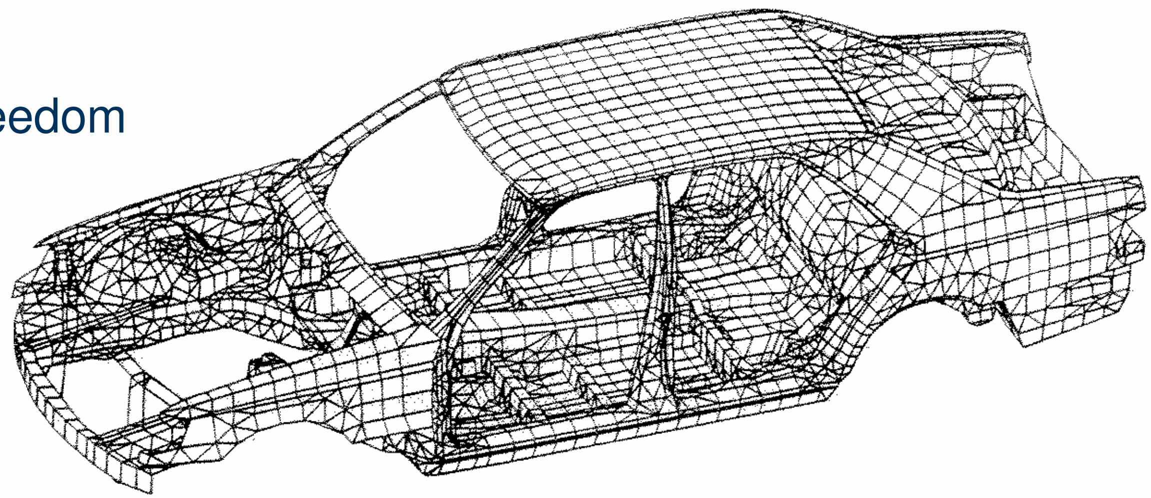
$$(K - \lambda_s M)\ \mu\varphi = M\varphi$$



The mid 1980s: car frames

- Reduction methods
- ~50,000 node points
- ~60,000 finite elements
- ~264,000 degrees of freedom
- 50 eigenvectors
- 2,505 CPU seconds
- 0.9 hours elapsed time
- 60 Mwords of memory
- 173 Mwords of disk space

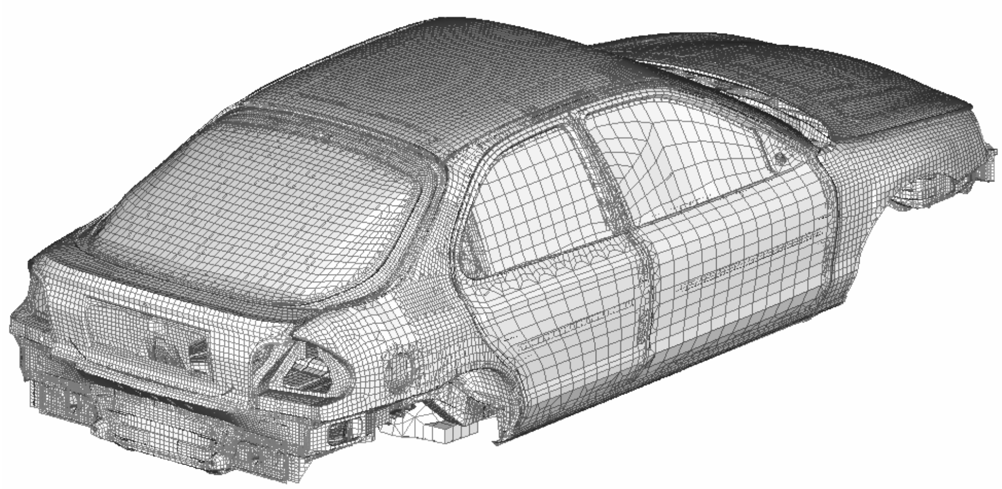
$$A = C^{-1}(M)\ C^{-T}; (K + \lambda_s M) = CC^T$$



The mid 1990s: full body models

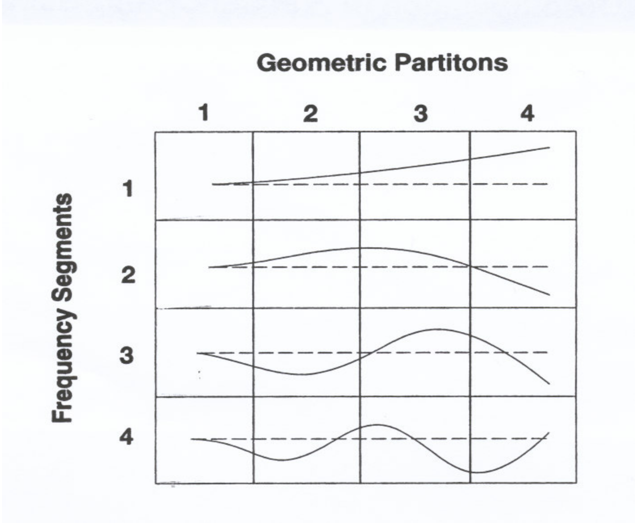
- Block, shifted Lanczos method
- ~ 275,000 elements,
- ~ 270,000 node points
- ~1.6 million degrees of freedom
- ~1,000 eigenvectors
- 4,936 CPU seconds
- 221 GBytes of I/O
- 1.7 hours elapsed time
- 128 MWords of memory
- 65 GBytes of disk used

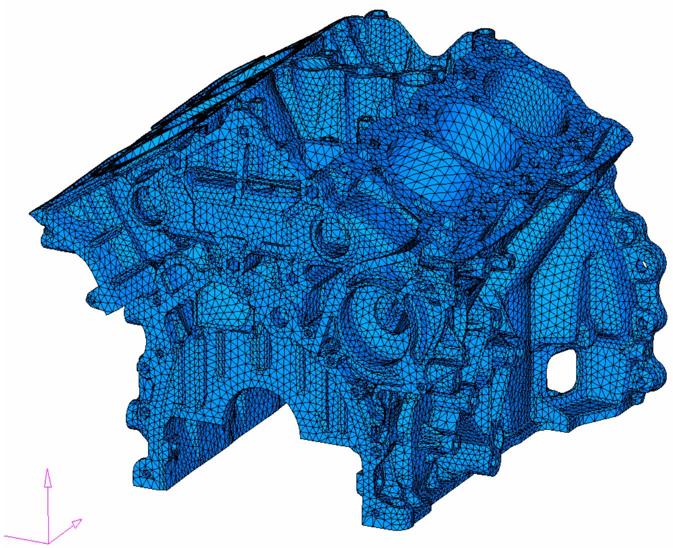
$$Q_{j+1}B_{j+1} = (K - \lambda_s M)^{-1} M Q_j - Q_j A_j - Q_{j-1} B_j^T$$



The mid 2000's: detailed models

- Domain decomposed solutions
- ~12 million node points
- ~7.2 million elements
- ~35 million degrees of freedom
- 20 eigenvectors
- ~16 GB memory used
- ~680 minutes of elapsed time
- ~100,000 CPU seconds
- ~11.5 Tera-bytes of I/O
- ~630 Giga-bytes of disk used





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- **Life-cycle simulation eigenvalue problems in NASTRAN**

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Simulations and solutions

- Rotational dynamics
  - Sequence of quadratic, asymmetric eigenvalue problems
  - Linearization and spectral transformation
- Vibro-acoustics
  - Quadratic, topologically unsymmetric eigenvalue problem
  - Symmetrization and reduced order Lanczos method
- Aero-elasticity
  - Sequence of complex, quadratic eigenvalue problems
  - Dynamic reduction and augmentation of modal space
- Fluid-structure interaction
  - Real, symmetric problem with dense mass
  - Virtual mass and transformed inner product Lanczos method

Rotational dynamic simulation

- Applications

  Turbines and power generators

  Car wheels and landing gears

  Windmills and rotating machinery

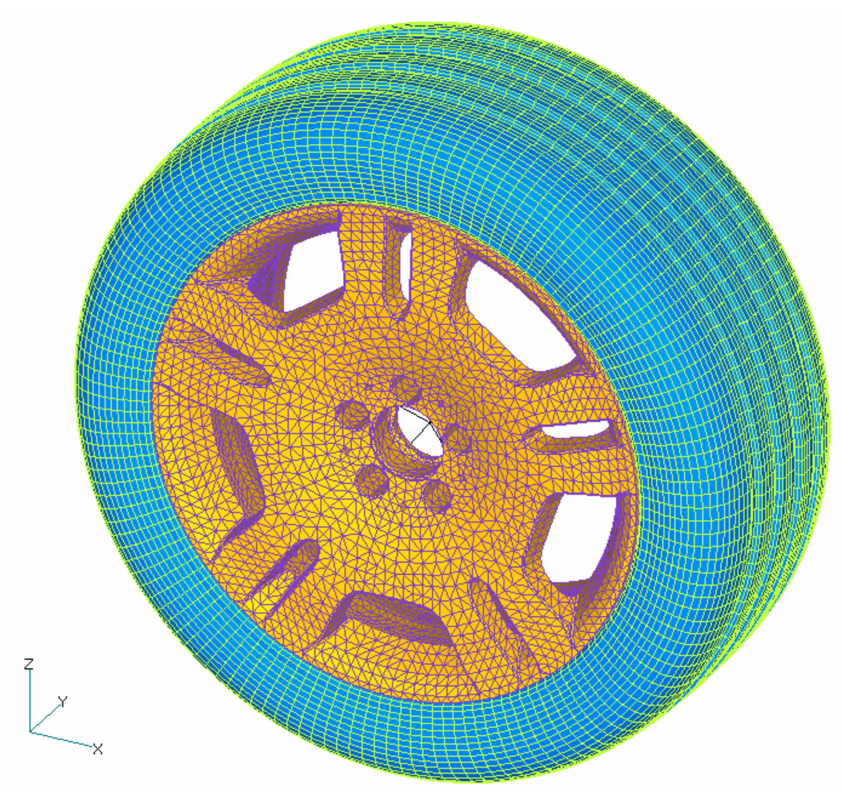
  Drive trains

- Analyses

  Whirl modes and critical speeds

  Regions of instability and resonance

  Frequency and transient response



Quadratic problem linearization

- Rotor dynamic equation of motion in fixed reference system

$$M \ddot{u} + (B + \Omega C) \dot{u} + (K + \Omega H) u = 0$$

- Block linear form

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \dot{u} \end{bmatrix} + \begin{bmatrix} \overline{B} & \overline{K} \\ -I & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ u \end{bmatrix} = 0$$

- Transformation to frequency domain

$$u = e^{\lambda t} \phi \qquad (\lambda^2 M + \lambda \overline{B} + \overline{K}) \phi = 0$$

- Unsymmetric eigenvalue problem

$$\begin{bmatrix} \dot{\psi} \\ \psi \end{bmatrix}^T (\lambda \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \overline{B} & \overline{K} \\ -I & 0 \end{bmatrix}) = 0 \qquad (\lambda \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \overline{B} & \overline{K} \\ -I & 0 \end{bmatrix}) \begin{bmatrix} \dot{\phi} \\ \phi \end{bmatrix} = 0$$

- Complex solutions $\lambda_j = \alpha_j \pm i \omega_j \qquad f_j = \frac{\omega_j}{2\pi} \qquad g_j = 2 \frac{\alpha_j}{\omega_j}$

Complex spectral transformation

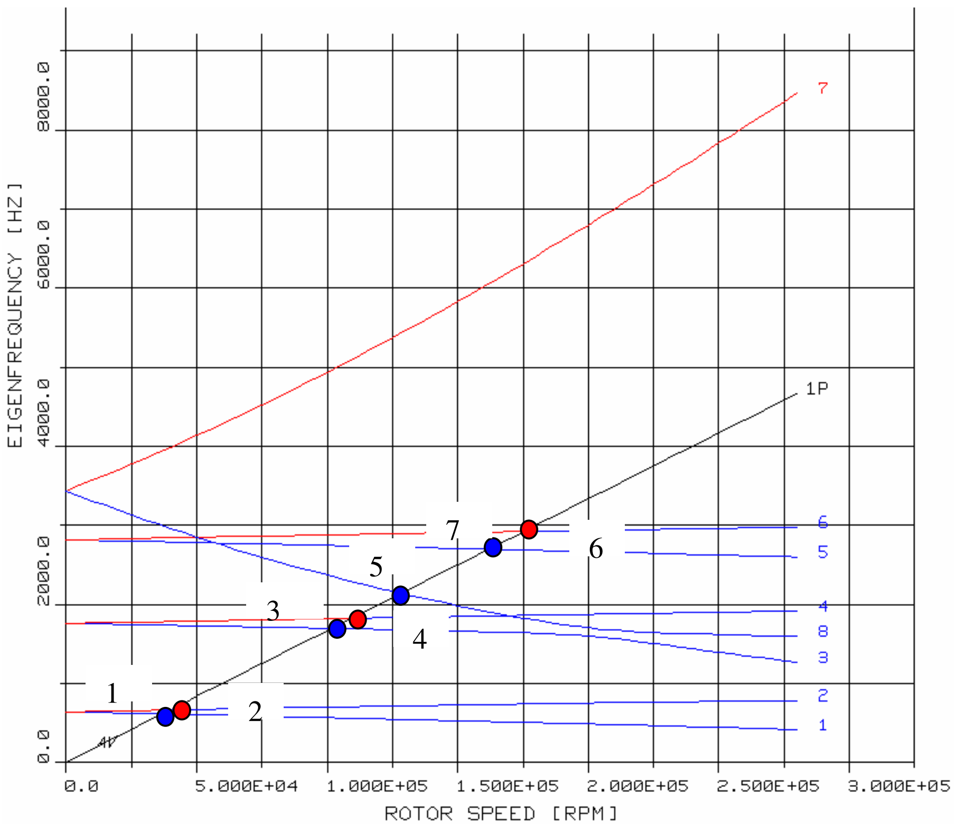
- Shifted eigenvalue $\lambda = \lambda_s + \mu$
- Shifted equation $$(\mu \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \overline{B} + M\lambda_s & \overline{K} \\ -I & \lambda_s I \end{bmatrix}) \begin{bmatrix} \dot{\phi} \\ \phi \end{bmatrix} = 0$$
- Inverted eigenvalue $$\overline{\lambda} = \frac{1}{\mu}$$
- Transformed problem $$(\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} + \overline{\lambda} \begin{bmatrix} \overline{B} + M\lambda_s & \overline{K} \\ -I & \lambda_s I \end{bmatrix}) \begin{bmatrix} \dot{\phi} \\ \phi \end{bmatrix} = 0$$
- Canonical form

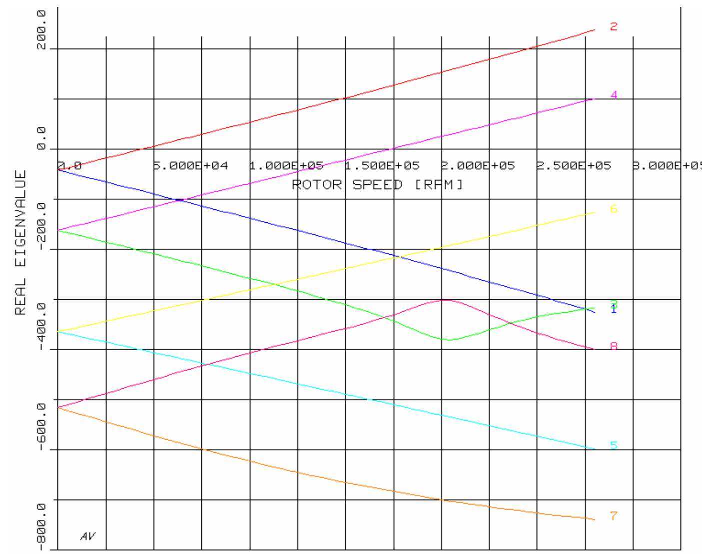
$$\overline{A}\,\overline{\phi}_j = \overline{\lambda}_j \overline{\phi}_j$$

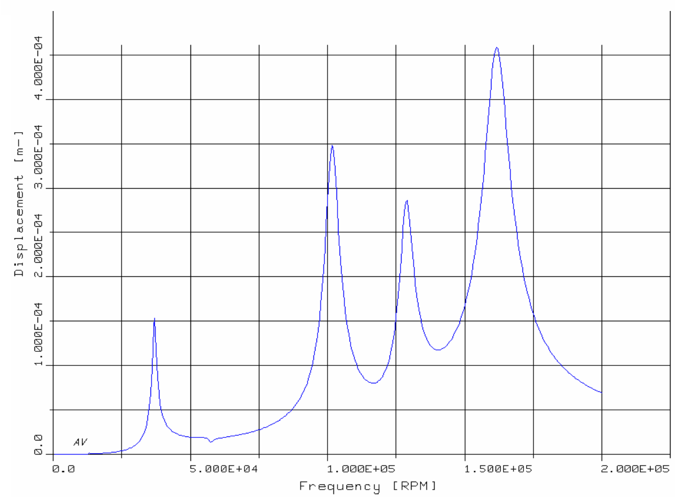
$$\overline{\psi}_j^H \overline{A} = \overline{\lambda}_j \psi_j^H$$

$$\overline{A} = \begin{bmatrix} -(\overline{B} + M\lambda_s) & -\overline{K} \\ I & -\lambda_s I \end{bmatrix}^{-1} \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}$$

Eigenvalue results in rotor dynamics







Resonances, instabilities, responses

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Vibro-acoustic analyses

- Applications

  Automotive vehicle interior noise

  Aero launch systems acoustics

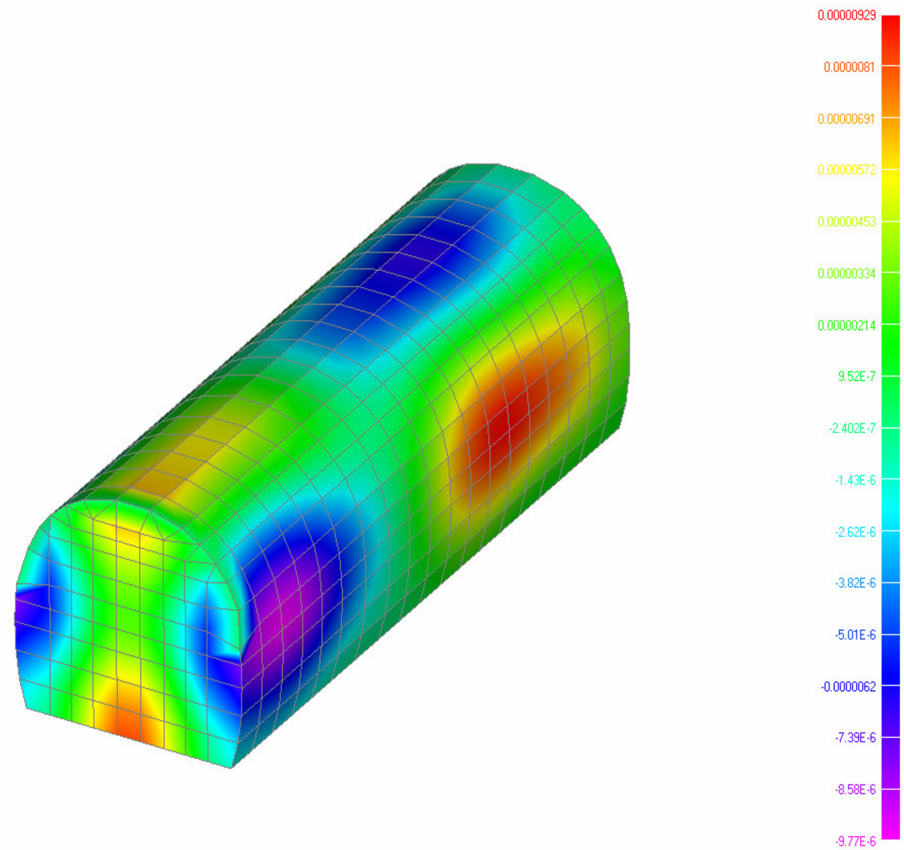
  Airplane cabin acoustics

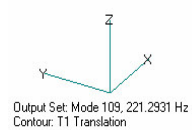
- Analysis components

  Find structure's interior surface

  Mesh interior volume

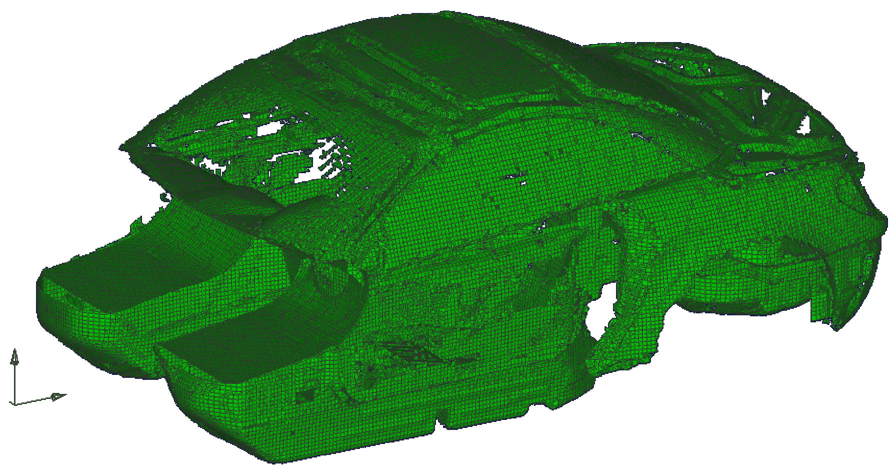
  Couple the surface loads





Vibro-acoustic coupling surface

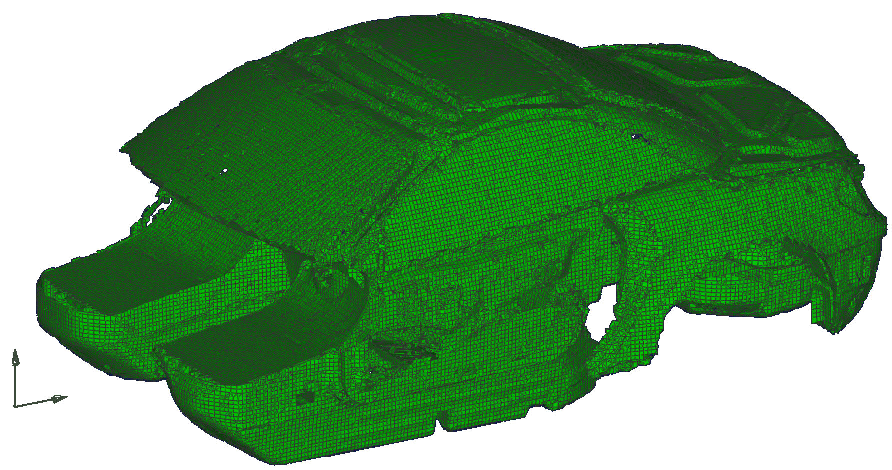
Missing on windshield and trunk lid



Still missing on side door

Normal tolerance = .2

Normal tolerance = .5



Coupled form symmetrization

- Vibro-acoustic equilibrium

$$\begin{bmatrix} M_s & 0 \\ A & M_f \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} K_s & -A^T \\ 0 & K_f \end{bmatrix}\begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- First equation

$$M_s\ddot{u} + (K_s + A^T M_f^{-1} A)u - A^T(p + M_f^{-1} Au) = 0$$

- Second equation

$$M_f K_f^{-1} M_f(\ddot{p} + M_f^{-1} A\ddot{u}) - Au + M_f(p + M_f^{-1} Au) = 0$$

- Balancing

$$q = p + M_f^{-1} Au$$

- Symmetric equation

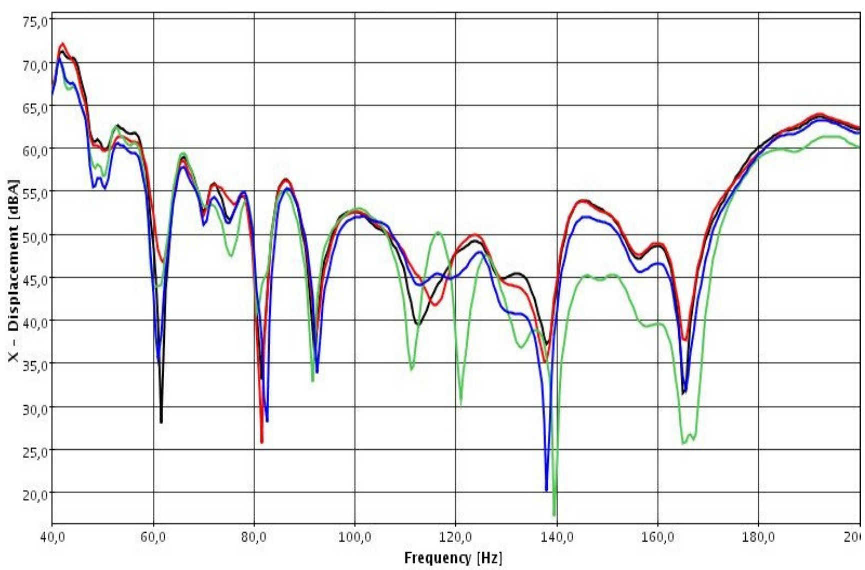
$$\begin{bmatrix} M_s & 0 \\ 0 & M_f K_f^{-1} M_f \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} K_s + A^T M_f^{-1} A & -A^T \\ -A & M_f \end{bmatrix}\begin{bmatrix} u \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Reduced order Lanczos method

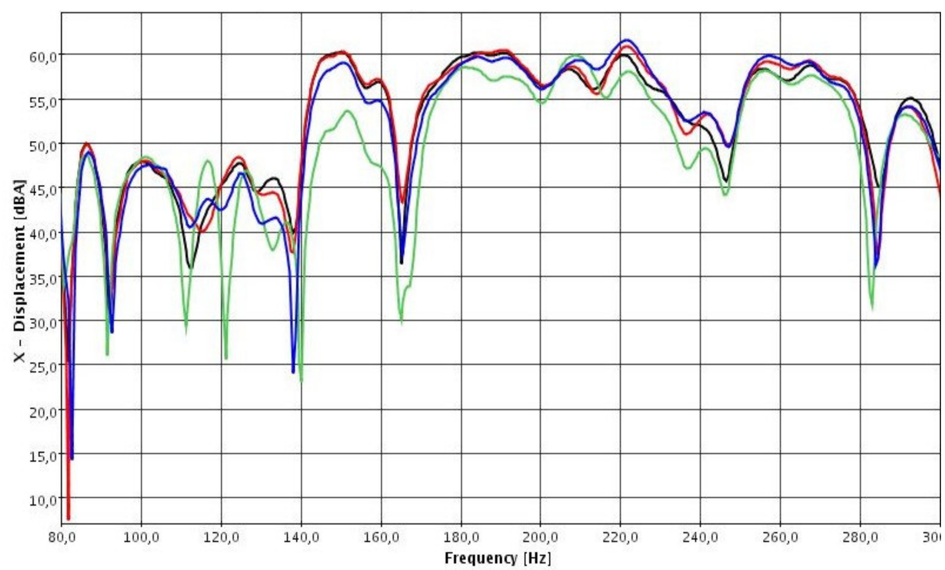
- Partitioning $\begin{bmatrix} P^T \\ P_0^T \end{bmatrix} M \begin{bmatrix} P & P_0 \end{bmatrix} = \begin{bmatrix} \overline{M} & 0 \\ 0 & 0 \end{bmatrix}$ $\overline{Q}_j = P^T Q_j$
- New operator $\overline{A} = (K - \lambda_s M)^{-1} P \overline{M}$ $\overline{Z}_j = \overline{A}\,\overline{Q}_j$
- New recurrence $\overline{Q}_{j+1} B_{j+1} = P^T \overline{Z}_j - \overline{Q}_j A_j - \overline{Q}_{j-1} B_j^T$
- Mass orthogonal $\overline{Q}_i^T \overline{M}\,\overline{Q}_j = Q_i^T P \overline{M} P^T Q_j = Q_i^T M Q_j = \delta_{ij}$
- Spectrum retained $\overline{Q}_j^T \overline{M} P^T \overline{A}\,\overline{Q}_j = Q_j^T M (K - \lambda_s M)^{-1} M Q_j = A_j$
- Reduced cost $\overline{Q}_{j+1} = \overline{Q}_{j+1} - \sum_i \overline{Q}_i (\overline{Q}_i^T \overline{M}\,\overline{Q}_{j+1})$
- Vector recovery $\varphi = \begin{bmatrix} P & P_0 \end{bmatrix} \begin{bmatrix} I \\ -(P_0^T K P_0)^{-1} (P_0^T K P) \end{bmatrix} \overline{\varphi}$

Acoustic analysis results

Location 1



Location 2



Benchmark
NORML = .2
NORML = .5
NORML = .75

Aero-elasticity

- Analyses

  Static aero-elasticity

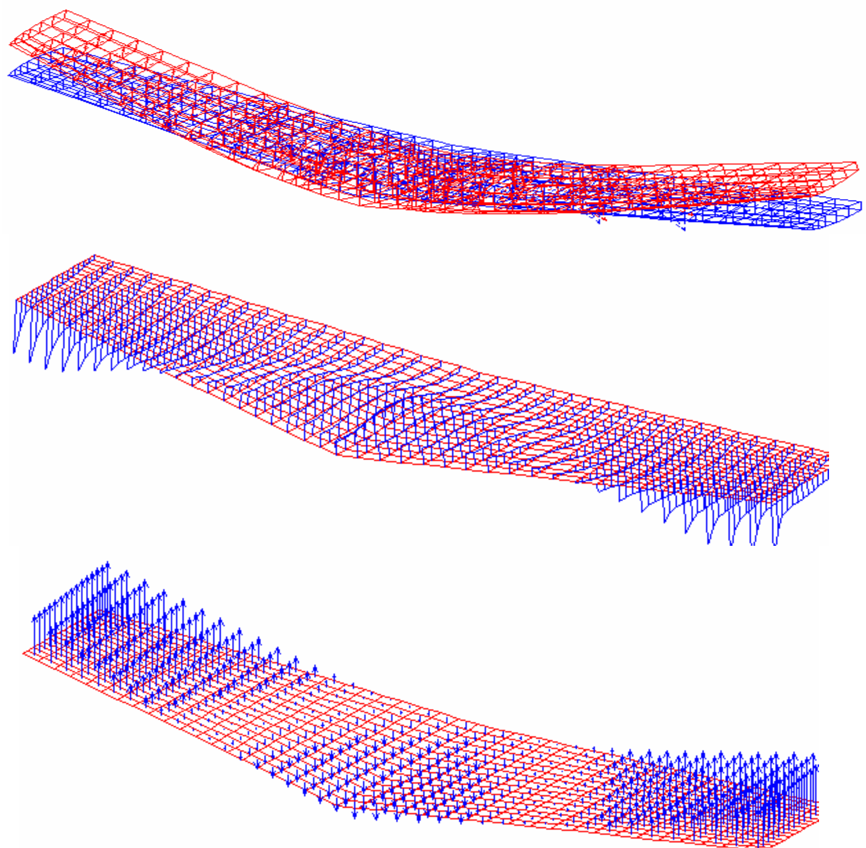
  Aeroelastic optimization

  Flutter analysis (effect of the unsteady, air-flow velocity dependent pressure distribution on the lifting surfaces)

- Aerodynamic theories

  Lifting, strip, piston and interference theories

  Subsonic and supersonic aerodynamic solutions



Dynamic reduction

- Analysis (boundary) and omitted (interior) partitions

$$M_{ff}\ddot{u}_f + B_{ff}\dot{u}_f + K_{ff}u_f = \begin{bmatrix} M_{oo} & M_{oa} \\ M_{ao} & M_{aa} \end{bmatrix} \begin{bmatrix} \ddot{u}_o \\ \ddot{u}_a \end{bmatrix} + \begin{bmatrix} B_{oo} & B_{oa} \\ B_{ao} & B_{aa} \end{bmatrix} \begin{bmatrix} \dot{u}_o \\ \dot{u}_a \end{bmatrix} + \begin{bmatrix} K_{oo} & K_{oa} \\ K_{ao} & K_{aa} \end{bmatrix} \begin{bmatrix} u_o \\ u_a \end{bmatrix} = 0$$

- Fixed boundary normal modes and static condensation modes

$$K_{oo}\Phi_{om} = M_{oo}\Phi_{om}\Lambda_{mm}$$

$$\Phi_{om}^T M_{oo}\Phi_{om} = I_{mm}$$

$$\begin{bmatrix} K_{oo} & K_{oa} \\ K_{ao} & K_{aa} \end{bmatrix} \begin{bmatrix} G_{oa} \\ I_a \end{bmatrix} = \begin{bmatrix} 0 \\ P_a \end{bmatrix} \quad G_{oa} = -K_{oo}^{-1}K_{oa}$$

- Transformation matrix and displacement transformation

$$T_{fh} = \begin{bmatrix} \Phi_{om} & G_{oa} \\ 0 & I_{aa} \end{bmatrix} \quad u_f = \begin{bmatrix} u_o \\ u_a \end{bmatrix} = \begin{bmatrix} \Phi_{om} & G_{oa} \\ 0 & I_{aa} \end{bmatrix} \begin{bmatrix} u_m \\ u_a \end{bmatrix} = T_{fh}u_h \quad Q_{hh} = \begin{bmatrix} 0 & 0 \\ 0 & Q_{aa} \end{bmatrix}$$

- Flutter problem in dynamically reduced space

$$M_{hh}\ddot{u}_h + B_{hh}\dot{u}_h + (K_{hh} - \tfrac{1}{2}\rho v^2 Q_{hh}(m,k))u_h = 0 \quad u_h = e^{i\omega t}\phi_h$$

$$(-\omega^2 M_{hh} + i\omega B_{hh} + K_{hh} - \tfrac{1}{2}\rho v^2 Q_{hh}(m,k))\phi_h = 0$$

Augmenting the modal space

- Full modal space $$\Lambda_{oo} = \Phi_{oo}^{T} K_{oo} \Phi_{oo}$$
- Complete flexibility $$K_{oo}^{-1} = \Phi_{oo} \Lambda_{oo}^{-1} \Phi_{oo}^{T}$$

- Incomplete modal space $$\Phi_{oo} = [\Phi_{om} \quad \Phi_{or}] \; ; m << o, m + r = o$$
- Component flexibilities $$K_{oo}^{-1} = \Phi_{om} \Lambda_{mm}^{-1} \Phi_{om}^{T} + \Phi_{or} \Lambda_{rr}^{-1} \Phi_{or}^{T} = Z_{m} + Z_{r}$$

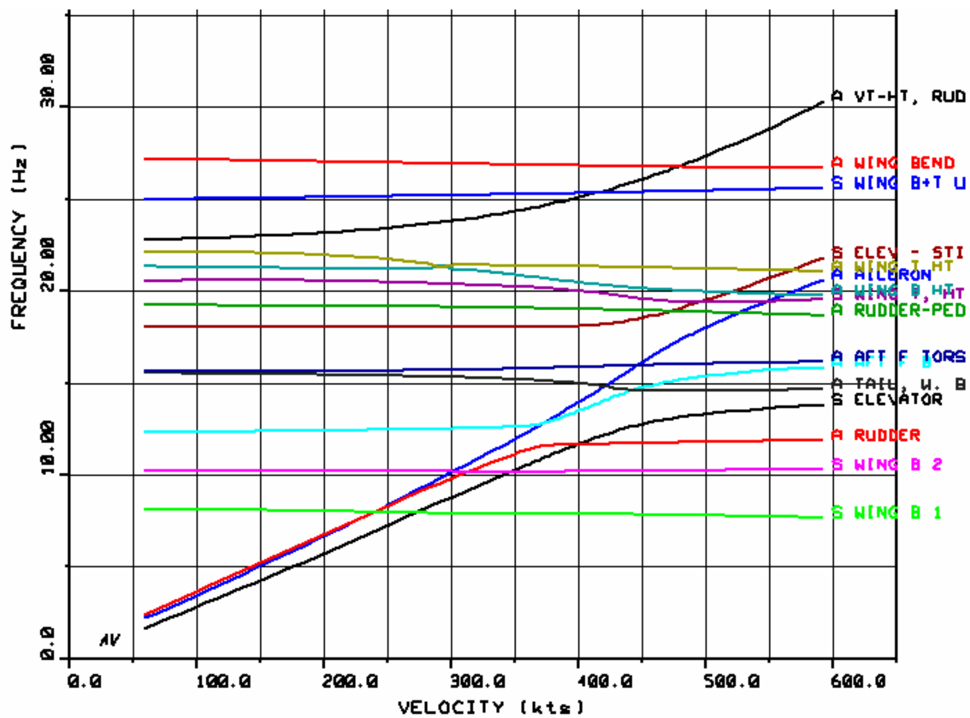
- Residual flexibility $$Z_{r} = K_{oo}^{-1} - \Phi_{om} \Lambda_{mm}^{-1} \Phi_{om}^{T}$$
- Residual vectors $$\Psi_{ol} = Z_r G_o ; P_o = G_o H_o(t)$$

- Augmented space $$\Phi_{o\overline{m}} = [\Phi_{om} \quad \Psi_{ol}] \; ; \overline{m} = m + l$$

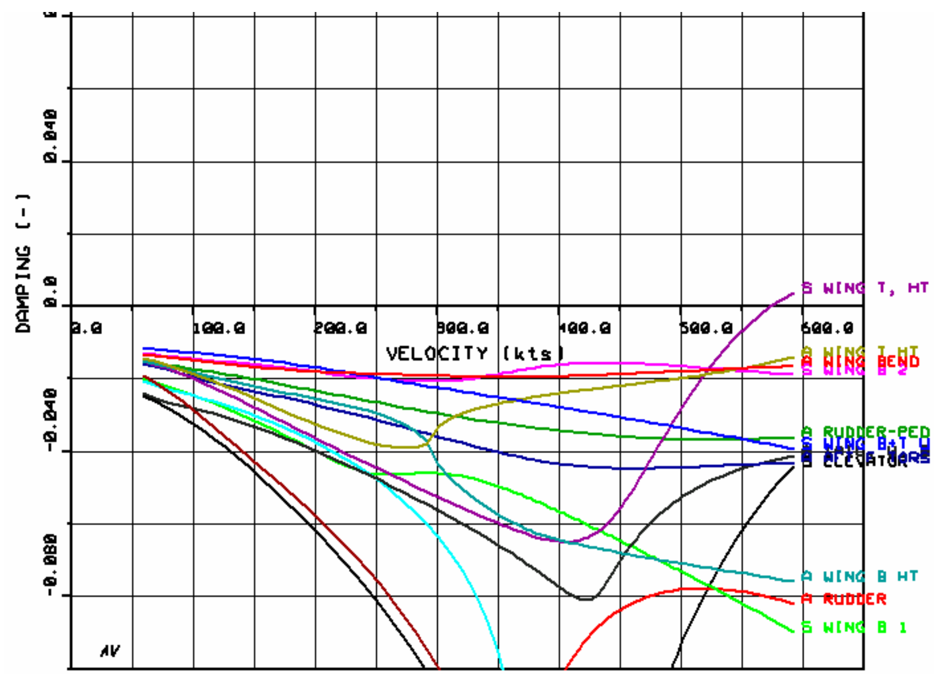
$$\Phi_{o\overline{m}}^{T} M_{oo} \Phi_{o\overline{m}} = I_{\overline{m}\,\overline{m}}$$

Eigenvalue results in flutter analysis

Frequency variation



Instability detection



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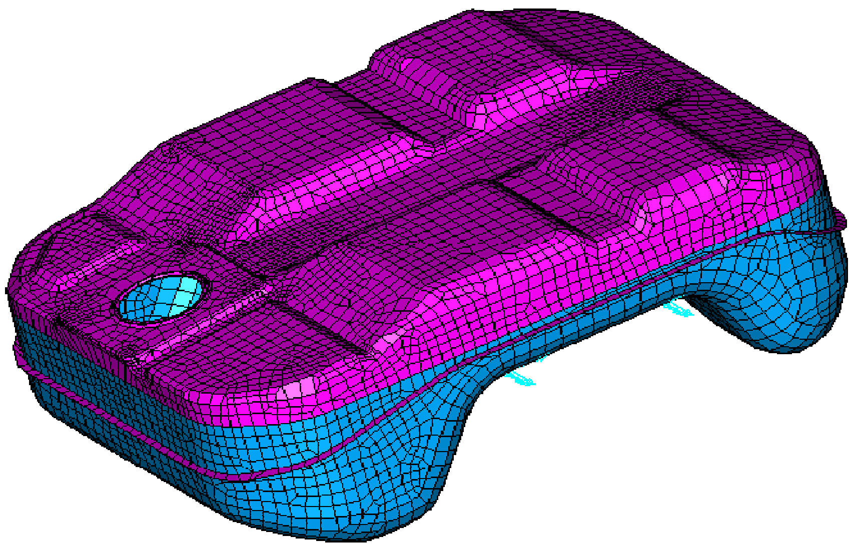
Fluid structure interaction

- For structures that vibrate in fluid
- Fluid contained in a structure

- Fuel tanks
- Off-shore structures in water

- Incompressible fluids
- Low frequencies

- Virtual mass matrix approach



Virtual mass matrix

- Coupled vibration with incompressible fluid

$$\begin{bmatrix} M_s & 0 \\ A & 0 \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} K_s & -A^T \\ 0 & K_f \end{bmatrix}\begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Eliminate pressure

$$A\ddot{u} + K_f p = 0; \quad p = -K_f^{-1} A\ddot{u}$$

- First equation

$$M_s\ddot{u} + K_s u - A^T p = M_s\ddot{u} + K_s u + A^T K_f^{-1} A\ddot{u} = 0$$

- Introducing virtual mass

$$M_v = A^T K_f^{-1} A$$

- Symmetric equation of coupled equilibrium

$$(M_s + M_v)\ddot{u} + K_s u = 0$$

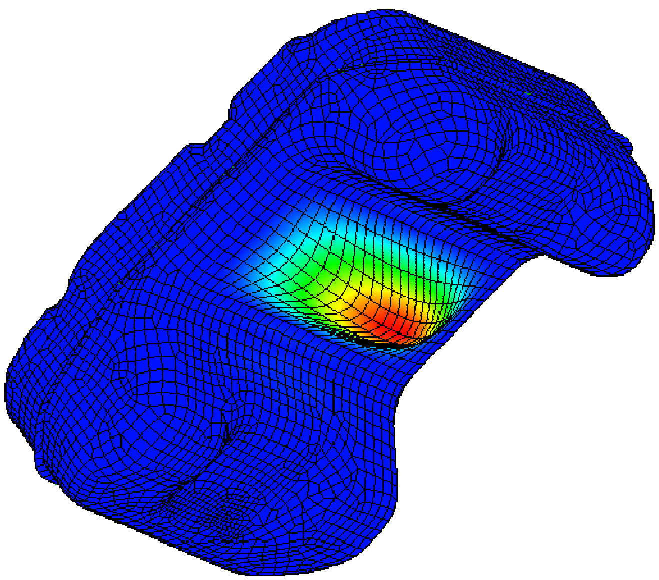
Transformed inner product Lanczos

- Factorize mass $M = CC^T;\ \overline{Q}_j = C^T Q_j$
- Transformed operator $\overline{A} = (K - \lambda_s M)^{-1} C$
- Transformed solution step $Z_j = \overline{A}\,\overline{Q}_j; \overline{Q}_j = C^T Q_j$
- Lanczos recurrence step $\overline{Q}_{j+1} B_{j+1} = C^T Z_j - \overline{Q}_j A_j - \overline{Q}_{j-1} B_j^T$
- Reduced cost $\overline{Q}_{j+1} = \overline{Q}_{j+1} - \sum_i \overline{Q}_i (\overline{Q}_i^T \overline{Q}_{j+1})$
- Spectrum retained

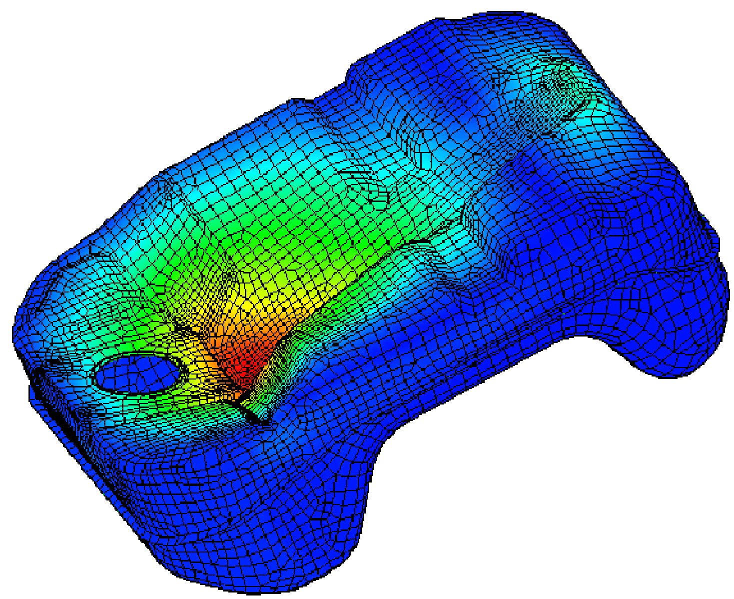
$$\overline{Q}_j^T C^T \overline{A}\,\overline{Q}_j = Q_j^T C C^T (K - \lambda_s M)^{-1} C C^T Q_j$$
$$= Q_j^T M (K - \lambda_s M)^{-1} M Q_j = A_j$$

Fluid-structure interaction results

With Fuel	No Fuel



Mode 1 – 7.5 Hz



Mode 1 – 27 Hz

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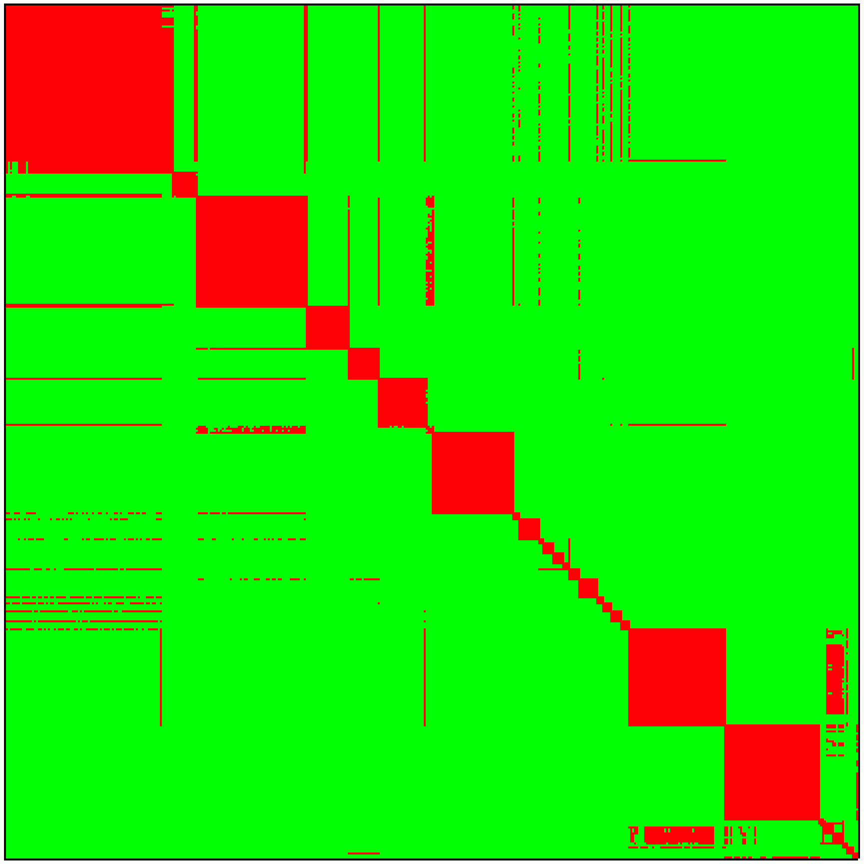
- **Conclusions**

    Statistics, state of the art and future predictions

Simulation matrix statistics

The constrained stiffness matrix of a current analysis problem

- Number of rows: 35,734,709
- Nonzero terms: 1,384,305,995
- Nonzero terms in sparse factor matrix: 43,827,004,000
- Memory used during factorization: 1,080,732,000 (4 byte) words
- Actual elapsed time of sparse factorization on a high performance workstation: 335 minutes

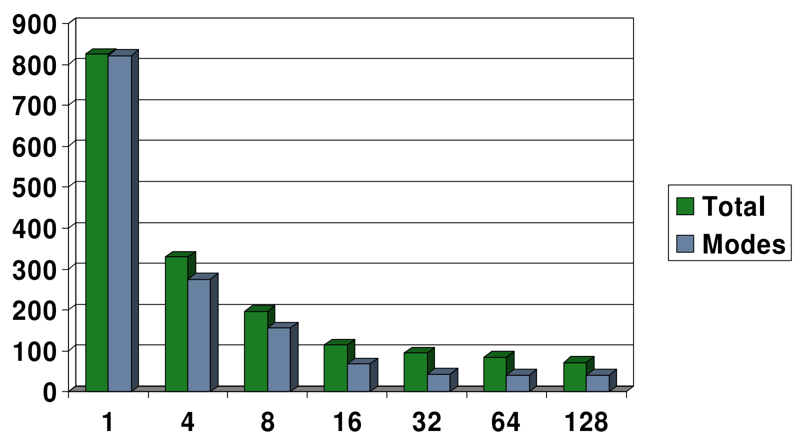


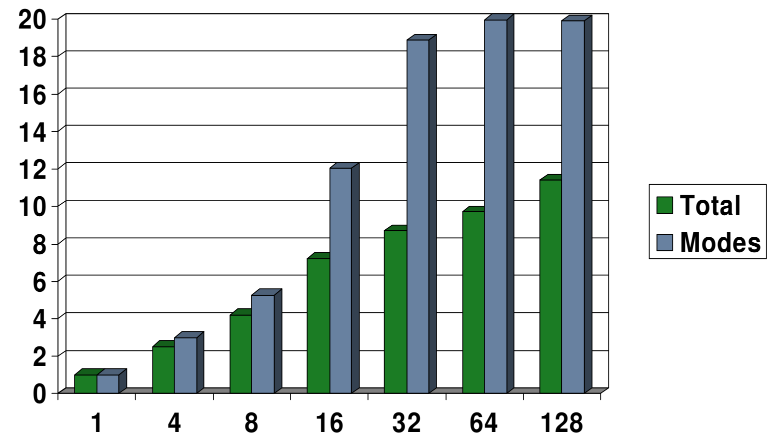
Computational technology statistics

NX NASTRAN

- Normal modes below 400 Hz
- ~268,000 node points
- ~275,000 finite elements
- 1.6 Million degrees of freedom
- Elapsed time and speed-up

- 64 node linux cluster
- Dual core (1.85 GHz) CPUs
- 50GB local SATA disks per node
- 4 GB memory per node
- GigE interconnect with HP MPI

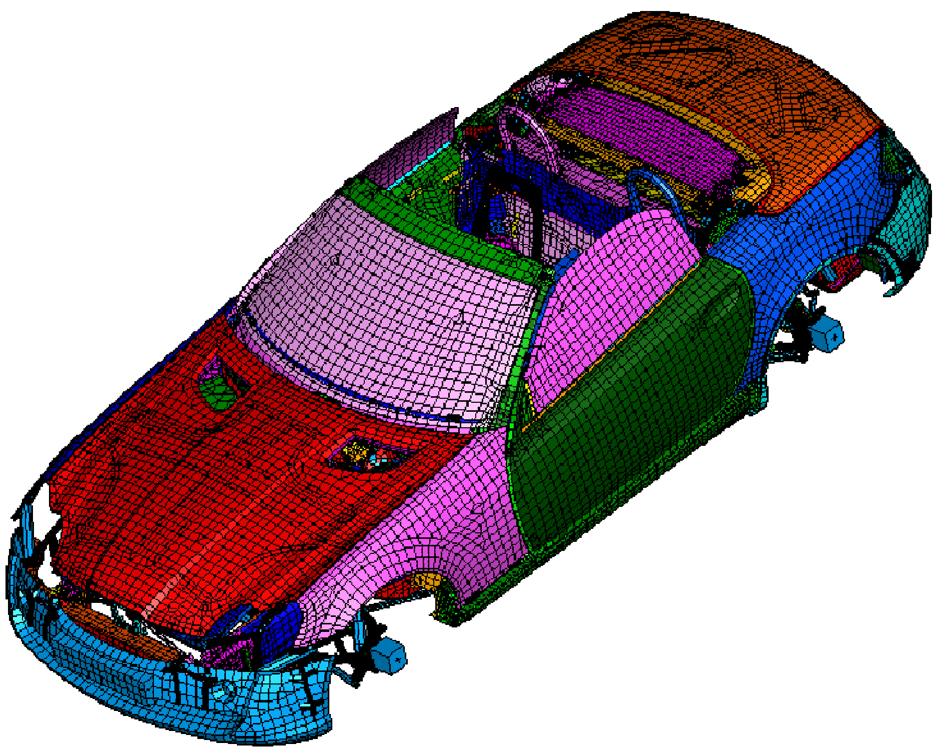




State of the art in life-cycle simulation

- Detailed multi-disciplinary solution
- ~35 million node points
- ~34 million elements
- ~200 million degrees of freedom
- 1000 Hz frequency range

- ~20 GB memory used
- ~420 minutes of elapsed time
- ~52,000 CPU seconds
- ~4.2 Tera-bytes of I/O
- ~980 Giga-bytes of disk used



Future trends

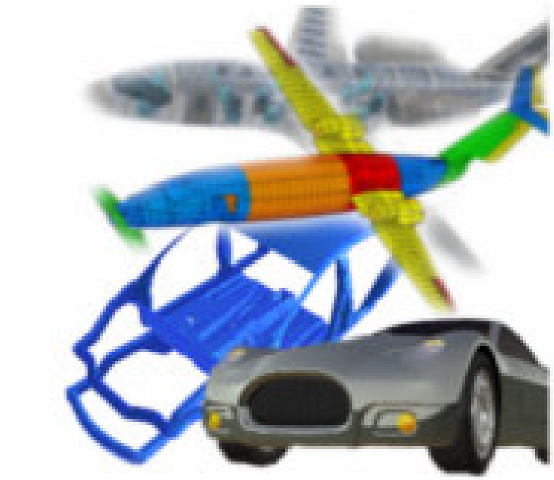
- **Accurate solution** of eigenvalue problems will continue to constitute a crucial component of life-cycle simulations

- Eigenvalue problem sizes continue to evolve and very likely to reach or even exceed the billion degree of freedom range: **Giga DOF problem**

- The ever shortening product development cycle time span in the industry require very highly scalable distributed techniques: **Peta-Scale computing**

Future requirements

- To extend the dominance of Lanczos method suggests more **physics sensitive adjustments** to the recurrence process for efficiency and accuracy reasons
- The coupled nature of life-cycle phenomena desires methods for **sequences of eigenvalue problems** or the possibility of parametric eigenvalue solutions
- The current **shifting and bounding technology** on the complex plane to establish regions containing clusters of eigenvalues needs significant improvement



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Thank You for your attention! Questions?